Exercise 1a)

$$\dfrac{\dfrac{\dfrac{a \vdash a}{a \vdash a \vee b}\,(\vee_{I_l})}{\vdash a \to (a \vee b)}\,(\to_I)}$$

Exercise 1b)

$$\dfrac{\dfrac{a \vee (b \wedge c) \vdash a \vee (b \wedge c) \qquad \dfrac{\dfrac{\dfrac{a \vdash a}{a \vdash a \vee b}\,(\vee I_l) \quad \dfrac{a \vdash a}{a \vdash a \vee c}\,(\vee I_l)}{a \vdash (a \vee b) \wedge (a \vee c)}\,(\wedge I)}{a \vee (b \wedge c), a \vdash (a \vee b) \wedge (a \vee c)}\,(W) \qquad \dfrac{\dfrac{\dfrac{\dfrac{b \wedge c \vdash b \wedge c}{b \wedge c \vdash b}\,(\wedge E_l) \quad \dfrac{b \wedge c \vdash b \wedge c}{b \wedge c \vdash c}\,(\wedge E_r)}{b \wedge c \vdash a \vee b}\,(\vee I_r) \quad \dfrac{b \wedge c \vdash b \wedge c}{b \wedge c \vdash a \vee c}\,(\vee I_r)}{b \wedge c \vdash (a \vee b) \wedge (a \vee c)}\,(\wedge I)}{a \vee (b \wedge c), b \wedge c \vdash (a \vee b) \wedge (a \vee c)}\,(W)}{a \vee (b \wedge c) \vdash (a \vee b) \wedge (a \vee c)}\,(\vee E)}{\vdash a \vee (b \wedge c) \to (a \vee b) \wedge (a \vee c)}\,(\to I)$$

Exercise 1c)

$$\dfrac{\dfrac{\dfrac{\exists x.\forall y.P(x,y) \vdash \exists x.\forall y.P(x,y) \qquad \dfrac{\dfrac{\dfrac{\dfrac{\forall y.P(b,y) \vdash \forall y.P(b,y)}{\exists x.\forall y.P(x,y), \forall y.P(b,y) \vdash \forall y.P(b,y)}\,(W)}{\exists x.\forall y.P(x,y), \forall y.P(b,y) \vdash P(b,a)}\,(\forall E)}{\exists x.\forall y.P(x,y), \forall y.P(b,y) \vdash \exists x.P(x,a)}\,(\exists I)}{\exists x.\forall y.P(x,y) \vdash \exists x.P(x,a)}\,(\exists E)}{\exists x.\forall y.P(x,y) \vdash \forall y.\exists x.P(x,y)}\,(\forall I)}{\vdash \exists x.\forall y.P(x,y) \to \forall y.\exists x.P(x,y)}\,(\to I)$$