Exercise 1a)

$$
\begin{prooftree}
\AxiomC{$a \vee (b \wedge c) \vdash a \vee (b \wedge c)$}
\AxiomC{$a \vdash a$}
\RightLabel{$(\vee I_l)$}
\UnaryInfC{$a \vdash a \vee b$}
\AxiomC{$a \vdash a$}
\RightLabel{$(\vee I_l)$}
\UnaryInfC{$a \vdash a \vee c$}
\RightLabel{$(\wedge I)$}
\BinaryInfC{$a \vdash (a \vee b) \wedge (a \vee c)$}
\RightLabel{(W)}
\UnaryInfC{$a \vee (b \wedge c), a \vdash (a \vee b) \wedge (a \vee c)$}
\AxiomC{$b \wedge c \vdash b \wedge c$}
\RightLabel{$(\wedge E_l)$}
\UnaryInfC{$b \wedge c \vdash b$}
\RightLabel{$(\vee I_r)$}
\UnaryInfC{$b \wedge c \vdash a \vee b$}
\AxiomC{$b \wedge c \vdash b \wedge c$}
\RightLabel{$(\wedge E_r)$}
\UnaryInfC{$b \wedge c \vdash c$}
\RightLabel{$(\vee I_r)$}
\UnaryInfC{$b \wedge c \vdash a \vee c$}
\RightLabel{$(\wedge I)$}
\BinaryInfC{$b \wedge c \vdash (a \vee b) \wedge (a \vee c)$}
\RightLabel{(W)}
\UnaryInfC{$a \vee (b \wedge c), b \wedge c \vdash (a \vee b) \wedge (a \vee c)$}
\RightLabel{$(\vee E)$}
\TrinaryInfC{$a \vee (b \wedge c) \vdash (a \vee b) \wedge (a \vee c)$}
\RightLabel{$(\rightarrow I)$}
\UnaryInfC{$\vdash a \vee (b \wedge c) \rightarrow (a \vee b) \wedge (a \vee c)$}
\end{prooftree}
$$

Exercise 1b)

$$
\begin{prooftree}
\AxiomC{$\exists x.\forall y.P(x,y) \vdash \exists x.\forall y.P(x,y)$}
\AxiomC{$\forall y.P(b,y) \vdash \forall y.P(b,y)$}
\RightLabel{(W)}
\UnaryInfC{$\exists x.\forall y.P(x,y), \forall y.P(b,y) \vdash \forall y.P(b,y)$}
\RightLabel{$(\forall E)$}
\UnaryInfC{$\exists x.\forall y.P(x,y), \forall y.P(b,y) \vdash P(b,a)$}
\RightLabel{$(\exists I)$}
\UnaryInfC{$\exists x.\forall y.P(x,y), \forall y.P(b,y) \vdash \exists x.P(x,a)$}
\RightLabel{$(\exists E)$}
\BinaryInfC{$\exists x.\forall y.P(x,y) \vdash \exists x.P(x,a)$}
\RightLabel{$(\forall I)$}
\UnaryInfC{$\exists x.\forall y.P(x,y) \vdash \forall y.\exists x.P(x,y)$}
\RightLabel{$(\rightarrow I)$}
\UnaryInfC{$\vdash \exists x.\forall y.P(x,y) \rightarrow \forall y.\exists x.P(x,y)$}
\end{prooftree}
$$

Exercise 2)

(**A1**) $a \rightarrow ((a \rightarrow a) \rightarrow a)$

(**A2**) $(a \rightarrow ((a \rightarrow a) \rightarrow a)) \rightarrow ((a \rightarrow (a \rightarrow a)) \rightarrow (a \rightarrow a))$

(**MP**) $(a \rightarrow (a \rightarrow a)) \rightarrow (a \rightarrow a)$

(**A1**) $a \rightarrow (a \rightarrow a)$

(**MP**) $a \rightarrow a$

(**A1**) $(a \rightarrow a) \rightarrow (b \rightarrow (a \rightarrow a))$

(**MP**) $b \rightarrow (a \rightarrow a)$

$a \vee \neg a$ normalizes to $\neg a \rightarrow \neg a$, which can be proven with the first five steps of this proof, using $\neg a$ instead of a in all instantiations.

(**Pr**) $\neg\neg a$

(**A1**) $\neg\neg a \rightarrow (\neg\neg\neg\neg a \rightarrow \neg\neg a)$

(**MP**) $\neg\neg\neg\neg a \rightarrow \neg\neg a$

(**A3**) $(\neg\neg\neg\neg a \rightarrow \neg\neg a) \rightarrow (\neg a \rightarrow \neg\neg\neg a)$

(**MP**) $\neg a \rightarrow \neg\neg\neg a$

(**A3**) $(\neg a \rightarrow \neg\neg\neg a) \rightarrow (\neg\neg a \rightarrow a)$

(**MP**) $\neg\neg a \rightarrow a$

(**MP**) a