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Handout 3: Specification and Verification Using Higher-Order Logic (summer term 2008)

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Exercise 1 Reasoning with Propositional Logic

In this exercise we prove some properties of propositional logic using only some basic rules.

The following rules may only be used: `notI, notE, conjI, conjE, disjI1, disjI2, disjE, impI, impE, mp, iffI, iffE, classical`.

These may be used together with the proof methods `rule`, `erule`, and `assumption`.

```
lemma I: "A --> A"

lemma "A & B --> B & A"

lemma "(A & B) --> (A | B)"

lemma "((A | B) | C) --> A | (B | C)"

lemma K: "A --> B --> A"

lemma "(A | A) = (A & A)"

lemma S: "(A --> B --> C) --> (A --> B) --> A --> C"

lemma "(A --> B) --> (B --> C) --> A --> C"

lemma "~ ~ A --> A"

lemma "A --> ~ ~ A"

lemma "(~ A --> B) --> (~ B --> A)"

lemma "((A --> B) --> A) --> A"

lemma "A | ~ A"

lemma "(~ (A & B)) = (~ A | ~ B)"
```

Exercise 2 Reasoning with Predicate Logic

In addition to the rules from the last exercise the following rules may be used for this exercise: `exI, exE, allI, allE`.

Prove the following lemmata or give a counterexample:

```
lemma "(EX x. ALL y. P x y) --> (ALL y. EX x. P x y)"

lemma "(ALL x. P x --> Q) = ((EX x. P x) --> Q)"

lemma "((ALL x. P x) & (ALL x. Q x)) = (ALL x. (P x & Q x))"

lemma "((ALL x. P x) | (ALL x. Q x)) = (ALL x. (P x | Q x))"

lemma "((EX x. P x) | (EX x. Q x)) = (EX x. (P x | Q x))"

lemma "(ALL x. EX y. P x y) --> (EX y. ALL x. P x y)"

lemma "(~ (ALL x. P x)) = (EX x. ~ P x)"
```