Prof. Dr. A. Poetzsch-Heffter
Peter Zeller, M. Sc.

TU Kaiserslautern
Fachbereich Informatik
AG Softwaretechnik

Sheet 13: Logik (SS 2017)

Bearbeitung in der Übung am 06./07. Juli

Aufgabe 1 Presburger Arithmethik

Let Σ_{PA} be the set of axioms of Presburger-arithmetic as defined in the lecture.

You may assume that: $\Sigma_{PA} \vdash_{\mathcal{F}} 0 + x = x \;\rightarrow\; 0 + (x + 1) = x + 1$

Use this to prove the following statement in the deductive system $\mathcal{F}$:

$$\Sigma_{PA} \;\vdash_{\mathcal{F}} \forall x.\, 0 + x = x$$

You can use the deduction theorem and the generalization theorem.

For writing down the proof you can use the same abbreviations as in exercise 3 on sheet 11.

Aufgabe 2 Herbrand-Expansion

Let $S = \left(\{a_{/0}, b_{/0}, n_{/0}, c_{/2}\}, \{e_{/2}\}\right)$ be a signature and $A \in FO(S)$ defined as:

$A \equiv \forall x.\forall y.\forall z.\; e\left(x, c(x, y)\right) \wedge \left(e(x, z) \rightarrow e\left(x, c(y, z)\right)\right) \wedge \neg e\left(a, c\left(b, c(a, n)\right)\right)$

a) Describe the domain used by Herbrand structures over the signature S.

b) Find formulas $A_1, \ldots, A_i$ (with $i \in \mathbb{N}_{\geq 1}$) from the Herbrand-expansion $E(A)$ of A, such that $A_1 \wedge \cdots \wedge A_i$ is unsatisfiable.

Aufgabe 3 Konsistenz und Erfüllbarkeit

Prove note 5.12 (b) *(which is Note 5.16 (b) in the English translation of the slides from 2014)*:
A first order theory T is satisfiable, if and only if it is consistent.

Aufgabe 4 Vollständigkeit und Konsistenz

Let Σ be a theory. Show that:
Σ is complete, if and only if there is no closed formula A, such that $\mathcal{T}_{\Sigma\cup\{A\}}$ and $\mathcal{T}_{\Sigma\cup\{\neg A\}}$ are both consistent.

Note:
With this you showed that completeness of a theory means, that it cannot be extended consistently in two different and contradicting ways.