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Sheet 10: Logik (SS 2017)

Abgabe: Freitag, 23. Juni, 15:30
Abgabekästen neben Raum 34-401.7 (bei AG Softwaretechnik)
Bitte geben Sie zu dritt ab.

The midterm exam is on Monday, 19.06.2017 at 19:00 in room 46-220.

To prepare for the exam it can be helpful to practice with tasks from old exams. You can find those at https://kai.cs.uni-kl.de/, which you can reach from the university network.

Aufgabe 1 Semantik der Prädikatenlogik

Let $A, B \in FO(S)$ be arbitrary formulas over a signature S. Use the semantics of predicate logic to show that: $(\exists x.\ A) \vee (\exists x.\ B) \models \exists x.\ A \vee B$

With that you have shown one part of equivalence 9 from Lemma 4.14.

Aufgabe 2 Substitution

a) Let $A \equiv \forall x. \left(\exists y.\ p\left(x, f(x,y)\right)\right) \to \left(\left(\forall x.\ p(x,y)\right) \wedge \left(\forall x.\ p(y,x)\right)\right)$

Assume $\theta = \left\{y / f(x,y)\right\}$ is a substitution. Calculate $A\theta$ (the application of substitution θ on formula A) following the definition from the lecture. Give all intermediate steps.

b) Give an example that shows: $(\forall x.\ A) \to A\left\{x/t\right\}$ would not be universally true for all formulas A, if our definition of applying a substitution would not rename bound variables (i.e. if for a quantifier Q applying the substitution had been defined as $(Qx.\ A)\theta := Qx.\ (A\theta)$ instead).

Aufgabe 3 Normalformen

a) Bring the following formulas into cleansed prenex normal form (CPF) and then transform them into a Skolem formula. Also submit your intermediate steps.

$$A_1 \equiv \forall x. \forall y.\ x < y \to \left(\exists z.\ x < z \wedge z < y\right)$$
$$A_2 \equiv \forall x_1. \exists x_2. \forall x_3. \exists x_4.\ p(x_1, x_2, x_3, x_4)$$
$$A_3 \equiv \left(\forall x.\ p(x)\right) \vee \left(\forall x. \exists y.\ q(x,y)\right)$$
$$A_4 \equiv \forall y.\ \left(\forall x.\ p(x,y)\right) \to \left(\forall x.\ p(y,x) \wedge \left(\exists y.\ q(y)\right)\right)$$

b) Let A_1' be your Skolem formula for formula A_1. Give a structure $\mathcal{M}$ and a variable assignment ψ, such that $\mathcal{B}_\psi^\mathcal{M}(A_1) \neq \mathcal{B}_\psi^\mathcal{M}\left(A_1'\right)$.

c) Let A_1' be your Skolem formula for formula A_1. Give a structure $\mathcal{M} = (D, I)$, which is a model for A_1 and for A_1'. In this structure $D = \mathbb{R}$ (the real numbers) and $I(<)(x,y) = x < y$ should hold.