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Sheet 9: Logik (SS 2017)

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Aufgabe 1 Bewertungen

Let $S = \left\{\left\{+_{/2}, *_{/2}\right\}, \left\{even_{/1}\right\}\right\}$ be a signature. We define the structure $\mathcal{M} = (D, I)$ over signature S with domain $D = \{0, 1, 2\}$. The functions + and − are interpreted by I according to the following table:

x	y	x + y	x*y
0	0	0	0
0	1	1	0
0	2	2	0
1	0	1	0
1	1	2	1
1	2	0	2
2	0	2	0
2	1	0	2
2	2	1	1

Moreover $I(even)(x) = 0$ if and only if $x = 1$.

Answer questions a) - c) for formulas A_1 to A_5:

$$A_1 \equiv \forall z.\ z = x \rightarrow (\forall x.\ z = x)$$
$$A_2 \equiv \exists n.\ \forall x.\ x + n = x$$
$$A_3 \equiv \forall x.\ \exists y.\ x * y \neq x$$
$$A_4 \equiv \exists z.\ \forall x.\ x \neq z \rightarrow \left(\exists y.\ x * y \neq x\right)$$
$$A_5 \equiv \exists x.\ \forall y.\ even(x + y)$$

a) Which variables are bound and which variable occur freely in the formula?

b) Is the formula closed?

c) What is the result of the valuation $\mathcal{B}_{\psi}^{\mathcal{M}}(A_i)$, if ψ is the variable assignment with $\psi(x) = 1$ for all variables x?

Aufgabe 2 Semantik

For each of the formulas below: Give one structure such that the formula is not satisfied and another structure which satisfies the formula (i.e. a model for the formula).

a) $\big(\forall y.\exists x.\ p(x,y)\big) \leftrightarrow \big(\exists x.\forall y.\ p(x,y)\big)$

b) $\forall y.\ \big(\exists x.\ p(x) \rightarrow q(y)\big) \rightarrow \Big(\big(\exists x.\ p(x)\big) \rightarrow q(y)\Big)$

Aufgabe 3 Modellierung

In this task we consider a data domain consisting of sets and natural numbers. Moreover you are given the following predicates and functions:

Predicate	Meaning
set(x)	true iff x is a set
number(x)	true iff x is a natural number
$x \in S$	true iff. S is a set and x is an element of S
$A \subseteq B$	true iff A and B are sets and A is a subset of B
finite(S)	true iff S is a finite set

Function	Meaning
$A \cup B$	Returns the union of sets A and B (or the empty set if A or B is not a set)
$A \cap B$	Returns the intersection of sets A and B (or the empty set if A or B is not a set)
$\{\}$	The empty set
$\{x\}$	The singleton set containing only x

Model the following statement in predicate logic using the predicates and functions defined above:

a) If all elements fro a set A are also elements of B, then A is a subset of B.

b) If there is an x, which is an element of set A, then $\{x\}$ is a subset of A.

c) For every set A there is a set A' which contains exactly the finite subsets of A.

d) There is no set, which contains itself as an element.

e) If A is not a subset of B, then there is an element in A which does not occur in B.

f) The intersection of two sets is empty, if and only if both sets have no element in common.