Prof. Dr. A. Poetzsch-Heffter
Peter Zeller, M. Sc.

TU Kaiserslautern
Fachbereich Informatik
AG Softwaretechnik

Sheet 8: Logik (SS 2017)

Abgabe: Freitag, 9. Juni, 15:30
Abgabekästen neben Raum 34-401.7 (bei AG Softwaretechnik)
Bitte geben Sie zu dritt ab.

Mid-term exam: The mid-term exam is on the 19.06.2017 at 19:00 in room 46-220. You can find more details on out webpage.

Aufgabe 1 Davis-Putnam-Verfahren

a) Use the Davis-Putnam method to check whether the following formula is a tautology:

$$\Big((p \wedge q) \vee (q \rightarrow r)\Big) \vee \neg(t \rightarrow \neg q \wedge \neg r) \vee (\neg p \wedge q)$$

b) Use the Davis-Putnam method to check whether the following formula is satisfiable:

$$\neg p \wedge (\neg t \vee q \vee p) \wedge (\neg s \vee \neg q) \wedge (s \vee \neg q \vee t) \wedge (r \vee \neg p) \wedge \neg r \wedge (\neg s \vee \neg q \vee s)$$

Aufgabe 2 Pure-Literal Regel im Davis-Putnam Verfahren

a) Show that the pure-literal rule is correct for formulas in negation normal form (NNF): If a variable $p \in V$ occurs only positively (i.e. without a negation directly in front of the variable) in a formula A (A in NNF), then $A[p/1]$ is satisfiable if and only if A is satisfiable.

The case for variables that occur only negatively is similar and does not have to be shown here.

b) Show that the statement in a) is not necessarily true, if A is not in NNF.

Aufgabe 3 Resolventenmethode

a) Use the resolution method to check if the following entailment holds:

$$p \rightarrow q, p \rightarrow r, q \leftrightarrow \neg r \models \neg p$$

b) Use the resolution method to check whether the following formula is satisfiable:

$$(\neg p \vee \neg q) \wedge (\neg r \vee \neg p \vee q) \wedge (q \vee r) \wedge (p \vee \neg q) \wedge (q \vee t) \wedge (\neg t \vee p \vee \neg q)$$

Aufgabe 4 Prädikatenlogik

In this task we consider a data domain consisting of natural numbers and lists of natural numbers. Moreover you are given the following predicates and functions:

Predicate	**Meaning**
list(x)	true iff x is a list
number(x)	true iff x is a natural number
$x < y$	true iff $x, y \in \mathbb{N}$ and $x < y$
$x \leq y$	true iff $x, y \in \mathbb{N}$ and $x \leq y$

Function	**Meaning**
length(l)	Returns the length of list l (or 0 if l is not a list)
elem(l, i)	Returns the element at position i in list l (or 0 if l is not a list or if i is not a natural number between 0 and $length(l) - 1$). The first element ist stored at position 0.
x + y	Returns the result of the addition (or 0 if $\{x, y\} \not\subseteq \mathbb{N}$)
x - y	Returns the result of the substraction (or 0 if $\{x, y\} \not\subseteq \mathbb{N}$ or $y > x$)
0	The number $0 \in \mathbb{N}$
1	The number $1 \in \mathbb{N}$

Model the following statement in predicate logic using the predicates and functions defined above:

a) The list l is sorted in ascending order.

b) There is a number which appears at least twice in list l.

c) The list l_1 contains all numbers which are also contained in list l_2 (it might contain more than that).

d) The list l_1 appears as a sublist in list $l2$.