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Sheet 7: Logik (SS 2017)

Bearbeitung in der Übung am 26./29./30. Mai
Beachten Sie die Ersatztermine!

Aufgabe 1 Tableaux: Lemma von Hintikka

Prove Hintikka's lemma (Slide 76):
Let $\Theta \subseteq F$ be complete[1]. Then Θ is satisfiable if and only if Θ is open[2].

In the proof you may assume:

- For every α-formula: $\mathcal{B}_\psi(\alpha) = min\left(\mathcal{B}_\psi(\alpha_1), \mathcal{B}_\psi(\alpha_2)\right)$
- For every β-formula: $\mathcal{B}_\psi\left(\beta\right) = max\left(\mathcal{B}_\psi\left(\beta_1\right), \mathcal{B}_\psi\left(\beta_2\right)\right)$

Hint: The lecture slides give the following idea for a proof:
If Θ is not open, then Θ is unsatisfiable by definition.
For the other case, let Θ be a complete and open set.
We define:

$$\psi(p) := \begin{cases} 0 & \neg p \in \Theta \\ 1 & \text{else.} \end{cases}$$

The variable assignment ψ is well-defined.
Use well-founded induction over the length of formulas, to show that $\mathcal{B}_\psi(A) = 1$ for all $A \in \Theta$.

Aufgabe 2 Disjunktive Normalform (DNF)

Give an equivalent formula in disjunctive normal form (DNF) for the following formula:

$$\neg a \wedge \left(\neg b \rightarrow (a \vee \neg c)\right)$$

To do this, first create a Tableaux for the formula. Then the DNF can be extracted from the Tableux by collecting the literals for each open branch (as shown in the lecture, slide 69).

[1] Here "complete" means that for all α-formulas α and alls β-formulas β:
$\alpha \in \Theta$ implies $\alpha_1 \in \Theta$ and $\alpha_2 \in \Theta$
$\beta \in \Theta$ implies $\beta_1 \in \Theta$ or $\beta_2 \in \Theta$
In the literature this property is also called "downward saturated".

[2] Here "open" means that a Variable p is in the set, than $\neg p$ must not be in the set.

Aufgabe 3 Negationsnormalform (NNF)

a) Transform the following formula into an equivalent formula in negation normal form (NNF):

$$\neg\Big(a \vee \big(b \wedge (b \to c)\big)\Big)$$

b) *(Additional exercise):* Prove the following statement:

For every formula $A \in F_{\{\neg,\wedge,\vee,\to\}}$ there is a formula $A' \in F_{\{\neg,\wedge,\vee\}}$ in NNF with $A \models\mid A'$ and $|A'| \leq 2 \cdot |A|$.

This is a variation of Lemma 3.13 from the lecture, slide 90.