Prof. Dr. A. Poetzsch-Heffter
Peter Zeller, M. Sc.

TU Kaiserslautern
Fachbereich Informatik
AG Softwaretechnik

Sheet 1: Logik (SS 2017)

Bearbeitung in der Übung am 20./21. April

Aufgabe 1 Diätplan

A 100 year old was asked “What is the secret of your long live?”. He replied “I follow a diet with strict rules: If I don’t drink beer with a meal, I always eat fish. Whenever I have fish with bear, I don’t eat icecream. If I eat icecream or if I don’t drink beer, I avoid having fish.”.

Formalize the diet plan with with propositional logic and try to find a more simple description.

Aufgabe 2 Strukturelle Induktion

Let $v(A)$ be the number of variable appearances in the propositional formula A, $k(A)$ the number of parentheses-pairs in A, $op(A)$ the number of operators in A and $bop(A)$ the number of binary operators in A.

These functions can be defined recursively over the structure of formulas, where $*$ denotes any binary operator from the set $\{\wedge, \vee, \rightarrow, \leftrightarrow\}$.

$$
\begin{array}{lll}
v(p_i) = 1 & v\big((\neg A)\big) = v(A) & v\big((A * B)\big) = v(A) + v(B) \\
k(p_i) = 0 & k\big((\neg A)\big) = 1 + k(A) & k\big((A * B)\big) = 1 + k(A) + k(B) \\
op(p_i) = 0 & op\big((\neg A)\big) = 1 + op(A) & op\big((A * B)\big) = 1 + op(A) + op(B) \\
bop(p_i) = 0 & bop\big((\neg A)\big) = bop(A) & bop\big((A * B)\big) = 1 + bop(A) + bop(B)
\end{array}
$$

Prove the following statements by using structural induction over the structure of propositional formulas:

a) Every propositional formula $A \in F$ contains the same number of parentheses-pairs and operators:

For all $A \in F$: $k(A) = op(A)$

Hint: We do not use the short notation here, which allows omitting parentheses. The statement is not true in that case.

b) Let n be the number of variable occurrences in $A \in F$. Then the number of operators in A is at least $n - 2$:

For all $A \in F$: $op(A) \geq v(A) - 2$

c) There is always one more variable occurence than binary operators in a formula.

For all $A \in F$: $v(A) = 1 + bop(A)$