3. Subtyping and Parametric Types

Overview:

- Typing of objects
- Subtype polymorphism
- Generic types for Java
- (Virtual types)

Motivation:

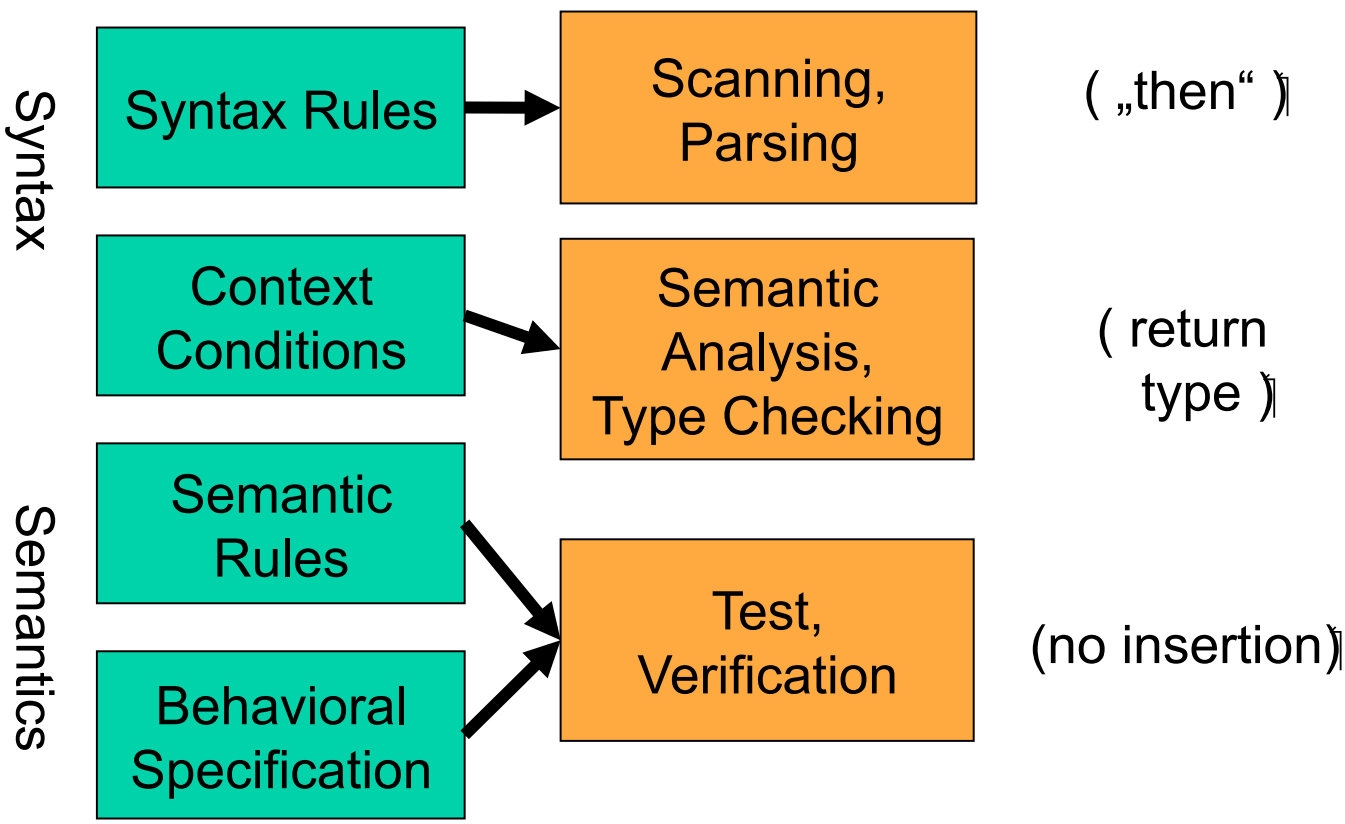
- Basic notions of type systems
- Typing as very light-weight specification
- Understanding subtyping
- Parametric polymorphism/generic types
- Alternative typing for OO programming

Correctness levels:

```
class ArraySet implements Set {
  private int[ ] array;
  private int next;
  …

  public void insert( int i ) {
    for ( int j = 0; j < next; j-- )
      if array[ j ] == i then return true;
    return false;
  }
}
```

Examples:



3.1 Typing of Objects

A type describes properties of values and objects (e.g.: int, Object, LinkedList<A>).

Typing assigns types to program elements – with slightly different meanings, e.g.:

- Expression (Ausdruck): Every evaluation of an expression of type T yields a value/object of type T.
- Variable: A variable of type T only holds values of type T.
- Procedures/methods: If the actual parameters have the types of the formal parameters, no type error occurs and the result is of the declared result type.
- The type of an object usually captures the information which messages the object understands. This way, typing can avoid „message not understood“-errors.

Remark:

Type systems are probably the most successful formal checking technique.

Explanation: (Typing)

The ***type system*** of a language *L* describes

- the types supported by *L*,
- the typing of *L*-programs (assignment of types to program elements),
- the ***type conditions*** (*Typisierungsbedingungen*) that programs have to fulfill.

A program satisfying the type conditions is called ***type correct*** (*typkorrekt*). Otherwise the program has ***typing errors*** (*Typisierungsfehler*).

A ***type error*** (*Typfehler*) occurs at runtime if

- a value/object of type T is assigned to a variable that has a type incompatible with T;
- an operation is called with parameters of types that are incompatible to the formal parameters.

A language is called ***type safe*** (*typsicher*) if the execution of type correct programs never leads to type errors. Type safety may depend on dynamic checks.

A type system is called ***sound*** (*sicher*) if it guarantees type safety.

Remark:

- Programs with typing errors need not cause type errors.
- In type safe languages, type correctness (static property) implies the absence of type errors (dynamic property).

□

Explanation: (Type checking)

Type checking consists of

- checking the type conditions at compile time (***static***),
- checking type properties at runtime (***dynamic***).

A language is called ***statically typed*** (*statisch typisiert*) if it does not use dynamic type checks.

□

Examples: (Dynamic type checks)

Java is not statically typed. It needs dynamic checks:

a) Type casts (Typkonvertierung):

```
Object obj = "Ich neues String-Objekt";
String str = (String) obj;
...
```

b) Array access (Feldzugriff):

```
String[] strfeld = new String[2];
Object[] objfeld = strfeld;
objfeld[0] = new Object();
int strl   = strfeld[0].length();
```

Invariant properties in type safe languages:

- Variables of type T always hold values/objects of type T (or of types compatible with T).
- The evaluation of expressions of type T always yields values/objects of type T (or of a compatible type).

Example: (Typing invariant)

In Java, typing ensures that accessing a field or invoking a method does not lead to field- or method-not-found exceptions.

Example: (Type system)

A good first example is the type system of Featherweight Java, FJ (see below).

□

Recommended literature:

Kim B. Bruce: Foundations of Object-Oriented Languages. Types and Semantics.

MIT Press, 2002.

Atsushi Igarashi, Benjamin Pierce, Philip Wadler: Featherweight Java: A Minimal Core Calculus for Java and GJ.

ACM Transactions on Programming Languages and Systems, 23:3, 2001.

T. Nipkow, D. v. Oheimb:
$Java_{light}$ is Type-Safe – Definitely.

Principles of Programming Languages, 1998.

3.2 Subtype Polymorphism

As discussed with Java, subtype polymorphism is obtained by a partial ordering relation on types:

Subtype relation:

Declaration: interface S extends T1, T2, ...
implicit: S < Object, S <T1, S <T2, ...

Declaration: class S extends T implements T1, T2, ...
implicit: S < T, S < T1, S < T2, ...

S < T implies: S[] < T[]

Subtype ordering is the reflexive and transitive closure.

Typing Problem in OO-Languages:

To enable substitutability, a subtype object must have

- all attributes of supertype objects
- all methods of supertype objects

Attributes and methods in the subtype must be ***compatible*** to those of the supertypes. Compatibility concerns typing and accessibility.

Explanation: (Co-/contravariance)

Let S be a subtype of T. Let TV be a type occurrence in T (attribute-, parameter-, result type) and SV the *corresponding* occurrence in S. We say:

- SV and TV are ***covariant*** if SV is a subtype of TV.
- SV and TV are ***contravariant*** if TV is a subtype of SV.

□

Fact:

OO-languages can only be statically typed iff:

- Attribute types are co- and contravariant, i.e. *invariant*.
- Parameter types are contravariant.
- Result types are covariant.

Example: (Co-/contravariance)

Assuming that overriding of attributes is allowed:

1. Attributes have to be covariant:

```
interface C1 { String a;  ... }
class D1 implements C1  { Object a; ... }
...
C1     cvar = new D1(); // initializes a
String svar = cvar.a;
   // Type error: a not covariant in D1
```

2. Attributes have to be contravariant:

```
interface C2 { Object a;  ... }
class D2 implements C2  { String a; ... }
...
C2     cvar = new D2();
cvar.a = new Object();
   // Type error: a not contravariant in D2
```

3. Parameter types have to be contravariant:

```
interface C3 { int m(Object p);  ... }
class D3 implements C3 {
    int m(String s){ s.length();...}
}
...
C3     cvar = new D3();
cvar.m( new Object() );
   // Type error when executing m:
   // parameter not contravariant
```

4. Result types have to be covariant:

```
interface C4 { String m();  ... }
class D4 implements C4 { Object m(){...} ...}
...
C4     cvar = new D4();
String svar = cvar.m();
   // Type error: result type not covariant
```

Covariance must as well hold for exception types.

□

Remark:

- Co- and contravariance are important to understand object-oriented typing and the principles underlying behavioral subtyping.
- Java:
  - essentially no contravariance for parameters
  - up to 1.4 no covariance for result types

□

3.3 Generic Types for Java

For many programming scenarios, subtype polymorphism is not the best choice. Often more precise type systems are desirable:

- Types with parameters
- Specialization of used types in subtypes

Disadvantages of subtype polymorphism:

- Specialization is only based on subtype order
- No parameterization w.r.t. the types used in a class
- No restriction on parameters

In addition to subtype polymorphism, most modern OO-languages also provide parametric polymorphism: e.g. C++, C#, Java 5, Eiffel, Ada 95.

We consider here Java. The examples are from:

G. Bracha, M. Odersky, D. Stoutamire, P. Wadler: Making the future safe for the past: Adding Genericity to the Java Programming Language. Object-Oriented Programming, Systems, Languages, and Applications, 1998.

M. Naftalin, P. Wadler: Java Generics. O'Reilly

Generic/parametric types:

Idea:

Allow type variables instead of type names at used occurrences of types in the program, i.e. in classes and interfaces.

Result: parametric class and interface declarations

Type variables are **declared** either

- together with class/interface name or
- together with method signature

Example: (Type variables)

```
class Pair<A,B>  {
    A fst;
    B snd;
    Pair(A f,B s){ ... }
    ...
}
```

`Pair<A,B>` is called a ***parametric/generic type***.

`Pair<_,_>` is called a ***type constructor***.

Type parameters are instantiated at object creation time. Thus, objects always have a "ground" type, i.e. a type without parameters:

```
Pair<String,Integer> pairvar =
      new Pair <String,Integer> (
            new String(), new Integer() );
```

A type in Java is represented by a *type expression*:

- a type identifier (without parameters),
- a type variable,
- a type constructor applied to type expressions.

Examples: (Type expressions)

- String where String is a declared type identifier.
- A where *A* is a declared variable.
- Pair<String, Object>
- Pair<Pair<A,A>, String>
- List< ? super Number >

Collection types and freely generated data types are the most important application of generic types.

Examples: (Use of generic types)

```
interface Collection<A> {
  public void add (A x);
  public Iterator<A> iterator ();
}


interface Iterator<B> {
  public B next ();
  public boolean hasNext ();
}


class NoSuchElementException
              extends RuntimeException {}


class LinkedList<C>
      implements Collection<C> {
   protected class Node {
      C elt;
      Node next = null;
      Node (C elt) { this.elt = elt; }
   }

   protected Node head = null, tail = null;

   public LinkedList () {}

// class continued on next slide
```

```
    public void add (C elt) {
       if (head == null) {
          head = new Node(elt);
          tail = head;
       } else {
          tail.next = new Node(elt);
          tail = tail.next;
       }
    }

    public Iterator<C> iterator () {
       return new Iterator<C> () {
          protected Node ptr = head;
          public boolean hasNext () {
             return ptr != null;
          }
          public C next () {
             if (ptr != null) {
                C elt = ptr.elt;
                ptr = ptr.next;
                return elt;
             } else {
                throw new
                  NoSuchElementException ();
             }
          }
       };
    }

}  // end of class LinkedList<C>
```

```
class Test {
  public static void main (String[] a) {
    LinkedList<String> ys =
                 new LinkedList<String>();
    ys.add("zero");
    ys.add("one");
    String y = ys.iterator().next();
  }
}
```

Generic methods:

Similar to type declarations, method declarations can as well be parameterized.

Example:

```
interface SomeCollection<A>
                 extends Collection<A> {
     public A some();
}

class Collections {
  public static <A,B> Pair<A,B> somepair(
                          SomeCollection<A> xa,
                          SomeCollection<B> xb ){
    return new Pair<A,B>(xa.some(),xb.some());
  }
}
```

Type parameters of methods are not instantiated at object creation time, but inferred at invocation sites.

New Features of Java 5:

Together with generic types other new features were introduced in Java 5:

- boxing and unboxing of primitive types
- new forms of loops (foreach)
- methods with a variable number of arguments
- enumerated types
- assertions

Example:

```
List<Integer> ints = Arrays.asList(1,2,3);
int s = 0;
for( int n : ints ) { s += n; }
assert s == 6;
```

Remark:

- Although the introduction of type parameters seems to be only a small extension, the resulting type system and its rules become quite complex.
- Generic types and methods are simple examples for generic software components.

Subtype Relation on Generic Types:

Subtype relation is declared or by wildcards:

```
LinkedList<A>  implements  Collection<A>
SomeCollection<A> extends  Collection<A>
Byte implements  Comparable<Byte>
<A   extends  Comparable<A>>
```

`List<Float>` is subtype of `List<? extends Number>`

The subtype relation between actual type parameters does not carry over to the instantiated types:

`LinkedList<String>` $\not\leq$ `LinkedList<Object>`

That is, type parameters are invariant.

Example: (Subtype relation)

The following example shows why equality is required for actual type parameters:

```
class Loophole {
  public static String loophole (Byte y) {
    LinkedList<String> xs =
                  new LinkedList<String>();
    LinkedList<Object> ys = xs;
                  // compilation error
    ys.add(y);
    return xs.iterator().next();
  }
}
```

Unlike with arrays, the types of the actual parameters for an object type are not available at runtime. Thus, a dynamic check is not possible.

Wildcard with “extends“:

```
interface Collection<E> {
   ...
   public boolean addAll(
          Collection<? extends E> c);
   ...
}
```

```
List<Number>  nums = new ArrayList<Number>();
List<Integer> ints = Arrays.asList(1,2);
List<Double>  dbls = Arrays.asList(2.7,3.1);
nums.addAll(ints);
nums.addAll(dbls);
assert nums.toString().equals(”[1,2,2.7,3.1]”);
```

Wildcards may appear in variable declarations:

```
List<? extends Number> nums2 = ints;
Number n = nums2.get(0);
nums2.add( 3 );          //compile-time error!
```

Wildcard with “super“:

The class Collections contains the following method:

```
public static <T> void copy(
                        List<? super T>  dst,
                        List<? extends T> src ){
   for( T elem: src ) {
      dst.add( elem );
}  }
```

Usage:

```
List<Object>  objs =
          Arrays.<Object>asList(2,3.1,”four”);
List<Integer> ints = Arrays.asList(5,6);
Collections.copy(objs, ints);
assert objs.toString().equals(”[5,6,four]”);
```

The type parameter can as well be given explicitly:

```
Collections.<Object>copy(objs, ints);
Collections.<Number>copy(objs, ints);
Collections.<Integer>copy(objs, ints);
```

Remark:

The Get and Put principle:

Use an extends wildcard when you only get values out of a structure,

use a super wildcard when you only put values into a structure,

don't use a wildcard when you both get and put.

Bounds for Type Parameters:

Java enables to restrict a type parameter *A*
at the declaration of A
by so-called ***bounds***. The bound requires that

A is a subtype of some given type *T*.

Bounds are declared together with the type variable:

<A extends T>

Example: (Bounded type parameters/classes)

```
import java.util.*;

public abstract
class SortableList
      <E extends Comparable<E>>
      extends AbstractList<E> {

  public void sort(){ /* ... */ }

  public boolean isSorted() {
    if( isEmpty() ) {
      return true;
    } else {
      Iterator<E> it = iterator();
      E elem1 = it.next();
      while( it.hasNext() ) {
        E elem2 = it.next();
        if( elem1.compareTo(elem2) > 0 ){
              return false;
        }
        elem1 = elem2;
      }
      return true;
    }
} }
```

Example: (Bounded type parameters/methods)

```
interface Comparable<A> {
   public int compareTo (A that);
}

class Byte implements Comparable<Byte> {
   private byte value;
   public Byte (byte value) {
     this.value = value;
   }
   public byte byteValue (){ return value; }
   public int compareTo (Byte that) {
     return this.value - that.value;
   }
}

class Collections {
  public static <A extends Comparable<A>>
    A max (Collection<A> xs) {
      Iterator<A> xi = xs.iterator();
      A w = xi.next();
      while (xi.hasNext()) {
        A x = xi.next();
        if (w.compareTo(x) < 0) w = x;
      }
      return w;
} }
```

Implementation of Generic Types in Java:

Parametric types can be translated by raw types. Such an implementation is based on:

- Parameter erasure:
  - Erase the type parameters
  - Use of type Object instead of type variables
  - Use of appropriate casts
- Bridge methods:
  - If a subclass instantiates a type variable of a superclass, it may be required to add further methods with an appropriate signature.

Example: (Erasure, bridge methods)

By means of parameter erasure and introduction of bridge methods, the above example results in the following:

```
interface Comparable {
  public int compareTo (Object that);
}

class Byte implements Comparable {
  private byte value;
  public Byte (byte value) {
    this.value = value;
  }
  public byte byteValue () {
    return value;
  }
  public int compareTo (Byte that) {
    return this.value - that.value;
  }
  public int compareTo (Object that) {
    return this.compareTo((Byte)that);
  }
}
```

```
class Collections {
 public static
 Comparable max(Collection xs) {
   Iterator xi = xs.iterator();
   Comparable w = (Comparable)xi.next();
   while (xi.hasNext()) {
     Comparable x = (Comparable)xi.next();
     if (w.compareTo(x) < 0) w = x;
   }
   return w;
 }
}
```

To handle type parameters that are only used in return types, Java 5 supports covariant overriding:

```
class Interval
      implements Iterator<Integer> {
  private int i, n;
  public Interval (int l, int u) {
     i=l; n=u; }
  public boolean hasNext () {
     return (i <= n); }
  public Integer next () {
     return new Integer(i++); }
}
```

Parameter erasure yields the following class where *next* has a more specific result type than *next* in Iterator:

```
class Interval implements Iterator {
  private int i, n;
  public Interval (int l, int u) {
     i=l; n=u; }
  public boolean hasNext () {
     return (i <= n); }
  public Integer next(){
     return new Integer(i++); }
}
```

Specification of Type Systems, Semantics and Soundness

Discussed based on the paper:

Atsushi Igarashi, Benjamin Pierce, Philip Wadler: *Featherweight Java: A Minimal Core Calculus for Java and GJ*, ACM Transactions on Programming, Languages and Systems, Vol. 23, No. 3, 2001

3.4 Virtual Types

Instead of parameterization of types, specialization can be used. This results in another kind of genericity:

```
class Vector {
  typedef ElemType as Object;
  void addElement( ElemType e ){ ... }
  ElemType elementAt{ int index ){ ... }
}

class PointVector extends Vector {
  typedef ElemType as Point;
}
```

`ElemType` is called a ***virtual*** type. In subclasses, virtual types can be overridden with subtypes.

In the following, we consider an extension of Java with virtual types.

Literature:

Kresten Krab Thorup: Genericity in Java with Virtual Types. European Conference on Object-Oriented Programming, 1997

Aspects of Virtual Types:

A. It is not necessary to distinguish between parametric types and „normal“ types:

```
interface IteratorOfObject {
  typedef A as Object;
  public A next();
  public boolean hasNext();
}

interface CollectionOfObject {
  typedef A as Object;
  typedef IteratorOfA as IteratorOfObject;
  public void add(A x);
  public IteratorOfA iterator();
}
```

```
class NoSuchElementException
      extends RuntimeException {}

class LinkedListOfObject
      implements CollectionOfObject {
  // inherits virtual types A, IteratorOfA
  protected class Node {
     A elt;
     Node next = null;
     Node (A elt) { this.elt = elt; }
  }
  protected Node head = null, tail = null;

  public LinkedListOfObject () {}

  public void add (A elt) {
     ... /* as on slide 3.16  */ }

  public IteratorOfA iterator () {
     return new IteratorOfA () {
       ... /* as on slide 3.16  */ };
  }
}
```

```
interface IteratorOfString
          extends IteratorOfObject {
  typedef A as String;
}

class LinkedListOfString
      extends  LinkedListOfObject {
  typedef A as String;
  typedef IteratorOfA as IteratorOfString;
}

class Test {
  public static void main (String[] a) {
    LinkedListOfString ys =
                new LinkedListOfString();
    ys.add("zero");
    ys.add("one");
    String y = ys.iterator().next();
  }
}
```

But each instantiation needs its own type declaration.

B. Subtype relations between generic types can be declared:

`LinkedListOfString` is subtype of `LinkedListOfObject` ;

That is, the following fragment is type correct:

```
LinkedListOfString strl =
            new LinkedListOfString();
LinkedListOfObject objl = strl;
objl.add( new Object() );
// VirtualTypeCastException
```

The covariance induced by virtual types is captured by dynamic type checking.

C. Recursive types and the connection with inheritance can be realised in a better way.

i) Mutual recursion illustrated by the observer pattern:

```
interface Observer {
  typedef SType as Subject;
  typedef EventType as Object;
  void notify (SType subj,EventType e);
}

class Subject {
  typedef OType as Observer;
  typedef EventType as Object;
  OType observers [];
  notifyObservers (EventType e) {
    int len = observers.length;
    for (int i =0;i <len;i++)↰
      observers [i ].notify(this,e);
  }
}


interface WindowObserver
          extends Observer {
  typedef SType as WindowSubject;
  typedef EventType as WindowEvent;
}

class WindowSubject extends Subject {
  typedef OType as WindowObserver;
  typedef EventType as WindowEvent;
  ...
}
```

ii) Inheritance with recursive occurrences of the declared type:

The declared type K can be designated with „This“ in the core of its declaration. In this case, all these occurrences are interpreted as occurrences of SUBK in a subclass SUBK, and respectively in further subtypes:

```
class SLinkedList {
   This tail;
   public This getTail() { return tail; }
}

class SLinkedListOfObject extends SLinkedList {
  typedef Elem as Object;
  Elem head;
  public Elem getHead() { return head; }
}

class SLinkedListOfString
      extends SLinkedListObject {
  typedef Elem as String;
  // SLinkedListOfString.getTail
  // yields SLinkedListOfString
}
```

Discussion of Virtual Types:

Advantages:

- A new relation between types is not necessary (only ”is subtype of”, not ”is type instance of”)
- Subtype relation between ”parametric” and ”instantiated” types is possible.
- Recursive dependencies can be handled in a more flexible way.

Disadvantages:

- Additional dynamic type checks are necessary (especially because of covariant method parameters):
  - Loss of static type checkability
  - Loss of efficiency
- Recursive instantiation (example Byte, slide 3.24) is not supported.

Remark:

- Thorup’s proposal allows the declaration of virtual types as subtypes of more than one type, and the distinction of a class type for the instantiation of objects.

- There are several approaches proposing an integration of parametric and virtual types, e.g.:
  K. Bruce, M. Odersky, P. Wadler:
  A statically safe alternative to virtual types.
  European Conference on Object-Oriented Programming, 1998.

Implementation of Virtual Types:

In the realization of the presented variant, type safety is achieved by introducing dynamic checking.

We distinguish between *primary* and *overriding* declarations of virtual types.

Essential ideas:

- For the realization of a virtual type T, use the type of the primary declaration of T.
- Define a cast method for each primary declaration of a virtual type T. This cast method is overridden in subtypes by overriding declarations.
- Use the cast method for the checking of actual parameters.
- In order to achieve type correctness in the resulting Java programs, introduce further casts.

Example: (Implementation of virtual types)

```
class Vector {
  typedef ElemType as Object;
  void addElement( ElemType e ) { ... }
  ...
}

class PointVector extends Vector {
  typedef ElemType as Point;
  void addElement( ElemType e ) {
    ...
    e.getX();
    ...
  }
}

...
PointVector pv;
Point pnt;
...
pv.addElement(pnt);
```

is transformed into

```
class Vector {
  Object cast$ElemType(Object o) {
    return o;
  }
  void check$addElement( Object o ) {
    this.addElement(
      this.cast$ElemType(o) );
  }
  void addElement( Object e ) { ... }
  ...
}

class PointVector extends Vector {
  Object cast$ElemType(Object o) {
    try {
      return (Point)o;
    } catch( ClassCastException c ) {
      throw
        new VirtualTypeCastException(...);
    }
  }
```

```
void addElement( Object e$0 ) {
    Point e = (Point)e$0;
        ...
    e.getX();
    ...
  }
}

...
PointVector pv;
Point pnt;
...
pv.check$addElement(pnt);
```