

3. Subtyping and Parametric Types

Overview:

- Typing of objects
- Subtype polymorphism
- A Generic type system
- Virtual types

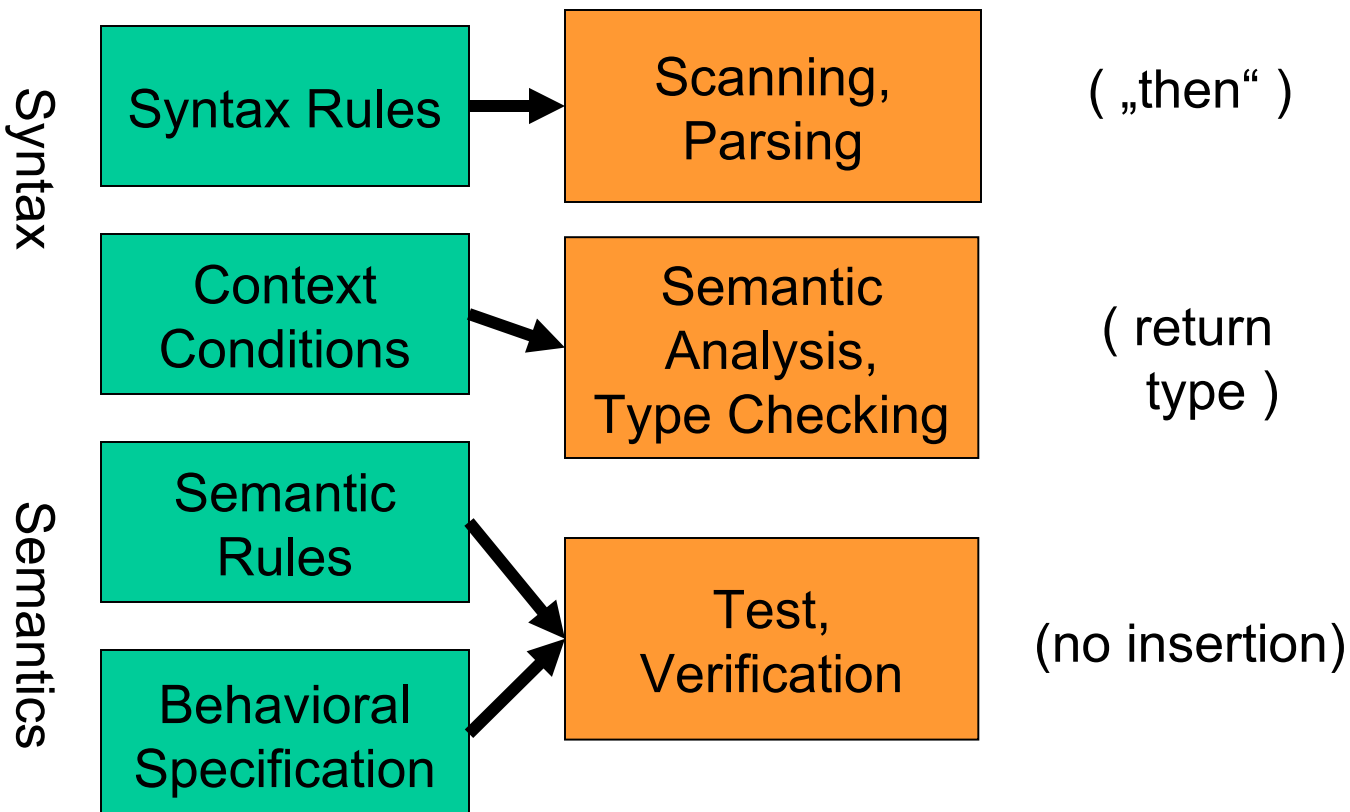
Motivation:

- Basic notions of type systems
- Typing as very light-weight specification
- Understanding subtyping
- Parametric polymorphism
- Alternative typing for OO programming

Correctness levels:

```
class ArraySet implements Set {  
  private int[ ] array;  
  private int next;  
  ...  
  
  public void insert( int i ) {  
    for ( int j = 0; j < next; j-- )  
      if array[ j ] == i then return true;  
    return false;  
  }  
}
```

Examples:



3.1 Typing of Objects

A type describes properties of values and objects (e.g.: int, Object, LinkedList<A>).

Typing assigns types to program elements with slightly different meanings, e.g.:

- Expression (Ausdruck): Every evaluation of an expression of type T yields a value/object of type T.
- Variable: A variable of type T only holds values of type T.
- Procedures/methods: If the actual parameters have the types of the formal parameters, no type error occurs and the result is of the declared type.
- The type of an object usually captures the information which messages the object understands. This way, typing can avoid „message not understood“-errors.

Remark:

Type systems are probably the most successful formal checking technique.



Explanation: (Typing)

The **type system** of a language L describes

- the types supported by L,
- the typing of L-programs (assignment of types to program elements),
- the **type conditions** (*Typisierungsbedingungen*) that programs have to fulfill.

A program satisfying the type conditions is called **type correct** (*typkorrekt*). Otherwise the program has **typing errors** (*Typisierungsfehler*).

A **type error** (*Typfehler*) occurs at runtime if

- a value/object of type T is assigned to a variable that has a type incompatible with T;
- an operation is called with parameters of types that are incompatible to the formal parameters.

A language is called **type safe** (*typsicher*) if the execution of type correct programs never leads to type errors. Type safety may depend on dynamic checks.

A type system is called **sound** (*sicher*) if it guarantees type safety.



Remark:

- Programs with typing errors need not cause type errors.
- In type safe languages, type correctness (static property) implies the absence of type errors (dynamic property). ■

Explanation: (Type checking)

Type checking consists of

- checking the type conditions at compile time (**static**),
- checking type properties at runtime (**dynamic**).

A language is called **statically typed** (*statisch typisiert*) if it does not use dynamic type checks. ■

Examples: (Dynamic type checks)

Java is not statically typed. It needs dynamic checks:

a) Type casts (Typkonvertierung):

```
Object obj = "Ich neues String-Objekt";  
String str = (String) obj;  
...
```

b) Array access (Feldzugriff):

```
String[] strfeld = new String[2];  
Object[] objfeld = strfeld;  
objfeld[0] = new Object();  
int strl      = strfeld[0].length();
```



Invariant properties in type safe languages:

- Variables of type T always hold values/objects of type T (or of types compatible with T).
- The evaluation of expressions of type T always yields values/objects of type T (or of a compatible type)

Example: (Typing invariant)

In StoJas, typing guarantees correct field access and existence of methods.



Example: (Type system)

A good first example is the type system of Featherweight Java, FJ (see citation below).



Recommended literature:

Kim B. Bruce: Foundations of Object-Oriented Languages. Types and Semantics.

MIT Press, 2002.

Atsushi Igarashi, Benjamin Pierce, Philip Wadler: Featherweight Java: A Minimal Core Calculus for Java and GJ.

ACM Transactions on Programming Languages and Systems, 23:3, 2001.

T. Nipkow, D. v. Oheimb:
Java_{light} is Type-Safe – Definitely.

Principles of Programming Languages, 1998.

3.2 Subtype Polymorphism

As discussed with Java, subtype polymorphism is obtained by a partial ordering relation on types:

Subtype relation:

Declaration: `interface S extends T1, T2, ...`

implicit: `S < Object, S < T1, S < T2, ...`

Declaration: `class S extends T implements T1, T2, ...`

implicit: `S < T, S < T1, S < T2, ...`

`S < T` implies: `S[] < T[]`

Subtype ordering is the reflexive and transitive closure.

Typing Problem in OO-Languages:

To enable substitutability, a subtype object must have

- all attributes of supertype objects
- all methods of supertype objects

Attributes and methods in the subtype must be **compatible** to those of the supertypes. Compatibility concerns **typing** and **accessibility**.

Explanation: (Co-/contravariance)

Let S be a subtype of T. Let TV be a type occurrence in T (attribute-, parameter-, result type) and SV the *corresponding* occurrence in S. We say:

- SV and TV are **covariant** if SV is a subtype of TV.
- SV and TV are **contravariant** if TV is a subtype of SV.



Fact:

OO-languages can only be statically typed iff:

- Attribute types are co- and contravariant, i.e. *invariant*.
- Parameter types are contravariant.
- Result types are covariant.

Example: (Co-/contravariance)

Assuming that overriding of attributes is allowed:

1. Attributes have to be covariant:

```
interface C1 { String a; ... }
class D1 implements C1 { Object a; ... }
...
C1 cvar = new D1(); // initializes a
String svar = cvar.a;
// Type error: a not covariant in D1
```

2. Attributes have to be contravariant:

```
interface C2 { Object a; ... }
class D2 implements C2 { String a; ... }
...
C2 cvar = new D2();
cvar.a = new Object();
    // Type error: a not contravariant in D2
```

3. Parameter types have to be contravariant:

```
interface C3 { int m(Object p); ... }
class D3 implements C3 {
    int m(String s){ s.length();...}
}
...
C3 cvar = new D3();
cvar.m( new Object() );
    // Type error when executing m:
    // parameter not contravariant
```

4. Result types have to be covariant:

```
interface C4 { String m(); ... }  
class D4 implements C4 { Object m(){...} ...}  
...  
C4 cvar = new D4();  
String svar = cvar.m();  
    // Type error: result type not covariant
```

Covariance must as well hold for exception types.



Remark:

- Co- and contravariance are important to understand object-oriented typing and the principles underlying behavioral subtyping.
- Java:
 - essentially no contravariance for parameters
 - up to 1.4 no covariance for result types



3.3 A Generic Type System

For many programming scenarios, subtype polymorphism is not the best choice. Often more precise type systems are desirable:

- Types with parameters
- Specialization of used types in subtypes

Disadvantages of subtype polymorphism:

- Specialization is only based on subtype order
- No parameterization w.r.t. the types used in class
- No restriction on parameters

In addition to subtype polymorphism, most modern OO-languages also provide parametric polymorphism: e.g. C++, C#, Java 5, Eiffel, Ada 95.

We consider here Java. The examples are from:

G. Bracha, M. Odersky, D. Stoutamire, P. Wadler:
Making the future safe for the past: Adding Genericity to the Java Programming Language. Object-Oriented Programming, Systems, Languages, and Applications, 1998.

M. Naftalin, P. Wadler: Java Generics. O'Reilly

Generic/parametric Types:

Idea:

Allow type variables instead of type names at used occurrences of types in the program, i.e. in classes and interfaces.

Result: parametric class and interface declarations

Type variables have to be declared either

- together with class/interface name or
- together with method signature

Example: (Type variables)

```
class Pair<A,B> {  
    A fst;  
    B snd;  
    Pair(A f,B s){ ... }  
    ...  
}
```



`Pair<A, B>` is called a ***parametric/generic type***.

`Pair<_, _>` is called a ***type constructor***.

Type parameters are instantiated at object creation time. Thus, objects always have a "ground" type, i.e. a type without parameters:

```
Pair<String,Integer> pairvar =  
    new Pair <String,Integer> (  
        new String(), new Integer() );
```

A type in Java is represented by a type expression:

- a type identifier (without parameters),
- a type variable,
- a type constructor applied to type expressions.

Examples: (Type expressions)

- String where String is a declared type identifier.
- A where A is a declared variable.
- Pair<String, Object>
- Pair<Pair<A,A>, String>
- List< ? super Number >



Collection types and freely generated data types are the most important application of generic types.

Examples: (Use of generic types)

```
interface Collection<A> {  
    public void add (A x);  
    public Iterator<A> iterator ();  
}
```

```
interface Iterator<B> {  
    public B next ();  
    public boolean hasNext ();  
}
```

```
class NoSuchElementException  
    extends RuntimeException {}
```

```
class LinkedList<C>  
    implements Collection<C> {  
  
    protected class Node {  
        C elt;  
        Node next = null;  
        Node (C elt) { this.elt = elt; }  
    }  
  
    protected Node head = null, tail = null;  
  
    public LinkedList () {}  
  
    // class continued on next slide
```

```

public void add (C elt) {
    if (head == null) {
        head = new Node(elt);
        tail = head;
    } else {
        tail.next = new Node(elt);
        tail = tail.next;
    }
}

public Iterator<C> iterator () {
    return new Iterator<C> () {
        protected Node ptr = head;
        public boolean hasNext () {
            return ptr != null;
        }
        public C next () {
            if (ptr != null) {
                C elt = ptr.elt;
                ptr = ptr.next;
                return elt;
            } else {
                throw new
                    NoSuchElementException ();
            }
        }
    };
}

} // end of class LinkedList<C>

```



```

class Test {
    public static void main (String[] a) {
        LinkedList<String> ys =
            new LinkedList<String>();
        ys.add("zero");
        ys.add("one");
        String y = ys.iterator().next();
    }
}

```



Generic methods:

Similar to type declarations, method declarations can as well be parameterized.

Example:

```

interface SomeCollection<A>
    extends Collection<A> {
    public A some();
}

class Collections {
    public static <A,B> Pair<A,B> somepair(
        SomeCollection<A> xa,
        SomeCollection<B> xb ) {
        return new Pair<A,B>(xa.some(), xb.some());
    }
}

```



Type parameters of methods are not instantiated at object creation time, but inferred at invocation sites.

New Features of Java 5:

Together with generic types other new features were introduced in Java 5:

- boxing and unboxing of primitive types
- new forms of loops (foreach)
- methods with a variable number of arguments
- enumerated types
- assertions

Example:

```
List<Integer> ints = Arrays.asList(1,2,3);  
int s = 0;  
for( int n : ints ) { s += n; }  
assert s == 6;
```



Remark:

- Although the introduction of type parameters seems to be only a small extension, the resulting type system and its rules become quite complex.
- Generic types and methods are simple examples for generic software components.



Subtype Relation on Generic Types:

Subtype relation is declared or by wildcards:

```
LinkedList<A> implements Collection<A>
```

```
SomeCollection<A> extends Collection<A>
```

```
Byte implements Comparable<Byte>
```

```
<A extends Comparable<A>>
```

```
List<Float> is subtype of List<? extends Number>
```

The subtype relation between actual type parameters does not carry over to the instantiated types:

```
LinkedList<String> ⊆ LinkedList<Object>
```

That is, type parameters are invariant.

Example: (Subtype relation)

The following example shows why equality is required for actual type parameters:

```
class Loophole {
    public static String loophole (Byte y) {
        LinkedList<String> xs =
            new LinkedList<String>();
        LinkedList<Object> ys = xs;
            // compilation error
        ys.add(y);
        return xs.iterator().next();
    }
}
```

Unlike with arrays, the types of the actual parameters for an object type are not available at runtime. Thus, a dynamic check is not possible.



Wildcard with “extends“:

```
interface Collection<E> {  
    ...  
    public boolean addAll(  
        Collection<? extends E> c);  
    ...  
}
```

```
List<Number>  nums = new ArrayList<Number>();  
List<Integer> ints = Arrays.asList(1,2);  
List<Double>  dbls = Arrays.asList(2.7,3.1);  
nums.addAll(ints);  
nums.addAll(dbls);  
assert nums.toString().equals("[1,2,2.7,3.1]");
```

Wildcards may appear in variable declarations:

```
List<? extends Number> nums2 = ints;  
Number n = nums2.get(0);  
nums2.add( 3 );           //compile-time error!
```

Wildcard with “super“:

The class `Collections` contains the following method:

```
public static <T> void copy(  
    List<? super T> dst,  
    List<? extends T> src ) {  
    for( T elem: src ) {  
        dst.add( elem );  
    }  
}
```

Usage:

```
List<Object> objs =  
    Arrays.<Object>asList(2,3.1,"four");  
List<Integer> ints = Arrays.asList(5,6);  
Collections.copy(objs, ints);  
assert objs.toString().equals("[5,6,four]");
```

The type parameter can as well be given explicitly:

```
Collections.<Object>copy(objs, ints);  
Collections.<Number>copy(objs, ints);  
Collections.<Integer>copy(objs, ints);
```

Remark:

The Get and Put principle:

Use an extends wildcard when you only get values out of a structure,

use a super wildcard when you only put values into a structure,

don't use a wildcard when you both get and put.



Bounds for Type Parameters:

Java enables to restrict a type parameter A by so-called **bounds**. The bound requires that

A is a subtype of some given type T .

Bounds are declared together with the type variable:

`<A extends T>`

Example: (Type bounds with classes)

```
import java.util.*;

public abstract
class SortableList
    <E extends Comparable<E>>
    extends AbstractList<E> {

    public void sort(){ /* ... */ }

    public boolean isSorted() {
        if( isEmpty() ) {
            return true;
        } else {
            Iterator<E> it = iterator();
            E elem1 = it.next();
            while( it.hasNext() ) {
                E elem2 = it.next();
                if( elem1.compareTo(elem2) > 0 ){
                    return false;
                }
                elem1 = elem2;
            }
            return true;
        }
    }
}
```



Example: (Type bounds with methods)

```
interface Comparable<A> {
    public int compareTo (A that);
}

class Byte implements Comparable<Byte> {
    private byte value;
    public Byte (byte value) {
        this.value = value;
    }
    public byte byteValue () { return value; }
    public int compareTo (Byte that) {
        return this.value - that.value;
    }
}

class Collections {
    public static <A extends Comparable<A>>
        A max (Collection<A> xs) {
        Iterator<A> xi = xs.iterator();
        A w = xi.next();
        while (xi.hasNext()) {
            A x = xi.next();
            if (w.compareTo(x) < 0) w = x;
        }
        return w;
    } }
}
```



Implementation of Generic Types in Java:

Parametric types can be translated by raw types. Such an implementation is based on:

- Parameter erasure:
 - Erase the type parameters
 - Use of type Object instead of type variables
 - Use of appropriate casts
- Bridge methods:
 - If a subclass instantiates a type variable of a superclass, it may be required to add further methods with an appropriate signature.

Example: (Erasure, bridge methods)

By means of parameter erasure and introduction of bridge methods, the above example results in the following:

```
interface Comparable {
    public int compareTo (Object that);
}

class Byte implements Comparable {
    private byte value;
    public Byte (byte value) {
        this.value = value;
    }
    public byte byteValue () {
        return value;
    }
    public int compareTo (Byte that) {
        return this.value - that.value;
    }
    public int compareTo (Object that) {
        return this.compareTo((Byte)that);
    }
}
```

```
class Collections {
    public static
    Comparable max(Collection xs) {
        Iterator xi = xs.iterator();
        Comparable w = (Comparable)xi.next();
        while (xi.hasNext()) {
            Comparable x = (Comparable)xi.next();
            if (w.compareTo(x) < 0) w = x;
        }
        return w;
    }
}
```



To handle type parameters that are only used in return types, Java 5 supports covariant overriding:

```

class Interval
    implements Iterator<Integer> {
private int i, n;
public Interval (int l, int u) {
    i=l; n=u; }
public boolean hasNext () {
    return (i <= n); }
public Integer next () {
    return new Integer(i++); }
}

```

Parameter erasure yields the following class where *next* has a more specific result type than *next* in *Iterator*:

```

class Interval implements Iterator {
private int i, n;
public Interval (int l, int u) {
    i=l; n=u; }
public boolean hasNext () {
    return (i <= n); }
public Integer next() {
    return new Integer(i++); }
}

```

3.4 Virtual Types

Instead of parameterization of types, specialization can be used. This results in another kind of genericity:

```
class Vector {  
    typedef ElemType as Object;  
    void addElement( ElemType e ){ ... }  
    ElemType elementAt( int index ){ ... }  
}
```

```
class PointVector extends Vector {  
    typedef ElemType as Point;  
}
```

`ElemType` is called a ***virtual*** type. In subclasses, virtual types can be overridden with subtypes.

In the following, we consider an extension of Java with virtual types.

Literature:

Kresten Krab Thorup: Genericity in Java with Virtual Types. European Conference on Object-Oriented Programming, 1997

Aspects of Virtual Types:

- A. It is not necessary to distinguish between parametric types and „normal“ types:

```
interface IteratorOfObject {
    typedef A as Object;
    public A next();
    public boolean hasNext();
}
```

```
interface CollectionOfObject {
    typedef A as Object;
    typedef IteratorOfA as IteratorOfObject;
    public void add(A x);
    public IteratorOfA iterator();
}
```

```

class NoSuchElementException
    extends RuntimeException {}

class LinkedListOfObject
    implements CollectionOfObject {
// inherits virtual types A, IteratorOfA
protected class Node {
    A elt;
    Node next = null;
    Node (A elt) { this.elt = elt; }
}
protected Node head = null, tail = null;

public LinkedListOfObject () {}

public void add (A elt) {
    ... /* as on slide 3.16 */ }

public IteratorOfA iterator () {
    return new IteratorOfA () {
        ... /* as on slide 3.16 */ };
}
}

```



```

interface IteratorOfString
    extends IteratorOfObject {
    typedef A as String;
}

class LinkedListOfString
    extends LinkedListOfObject {
    typedef A as String;
    typedef IteratorOfA as IteratorOfString;
}

class Test {
    public static void main (String[] a) {
        LinkedListOfString ys =
            new LinkedListOfString();
        ys.add("zero");
        ys.add("one");
        String y = ys.iterator().next();
    }
}

```

But each instantiation needs its own type declaration.

B. Subtype relations between generic types can be declared:

```
LinkedListOfString is subtype of  
                        LinkedListOfObject ;
```

That is, the following fragment is type correct:

```
LinkedListOfString str1 =  
                        new LinkedListOfString();  
LinkedListOfObject obj1 = str1;  
obj1.add( new Object() );  
// VirtualTypeCastException
```

The covariance induced by virtual types is captured by dynamic type checking.

C. Recursive types and the connection with inheritance can be realised in a better way.

i) Mutual recursion illustrated by the observer pattern:

```

interface Observer {
    typedef SType as Subject;
    typedef EventType as Object;
    void notify (SType subj,EventType e);
}

```

```

class Subject {
    typedef OType as Observer;
    typedef EventType as Object;
    OType observers [];
    notifyObservers (EventType e) {
        int len = observers.length;
        for (int i =0;i <len;i++)
            observers [i ].notify(this,e);
    }
}

```

```

interface WindowObserver
    extends Observer {
    typedef SType as WindowSubject;
    typedef EventType as WindowEvent;
}

```

```

class WindowSubject extends Subject {
    typedef OType as WindowObserver;
    typedef EventType as WindowEvent;
    ...
}

```

ii) Inheritance with recursive occurrences of the declared type:

The declared type K can be designated with „This“ in the core of its declaration. In this case, all these occurrences are interpreted as occurrences of SUBK in a subclass SUBK, and respectively in further subtypes:

```
class SLinkedList {
    This tail;
    public This getTail() { return tail; }
}
```

```
class SLinkedListOfObject extends SLinkedList {
    typedef Elem as Object;
    Elem head;
    public Elem getHead() { return head; }
}
```

```
class SLinkedListOfString
    extends SLinkedListObject {
    typedef Elem as String;
    // SLinkedListOfString.getTail
    // yields SLinkedListOfString
}
```

Discussion of Virtual Types:

Advantages:

- A new relation between types is not necessary (only "is subtype of", not "is type instance of")
- Subtype relation between "parametric" and "instantiated" types is possible.
- Recursive dependencies can be handled in a more flexible way.

Disadvantages:

- Additional dynamic type checks are necessary (especially because of covariant method parameters):
 - Loss of static type checkability
 - Loss of efficiency
- Recursive instantiation (example Byte, slide 3.24) is not supported.

Remark:

- Thorup's proposal allows the declaration of virtual types as subtypes of more than one type, and the distinction of a class type for the instantiation of objects.

- There are several approaches proposing an integration of parametric and virtual types, e.g.:
K. Bruce, M. Odersky, P. Wadler:
A statically safe alternative to virtual types.
European Conference on Object-Oriented Programming, 1998.



Implementation of Virtual Types:

In the realization of the presented variant, type safety is achieved by introducing dynamic checking.

We distinguish between *primary* and *overriding* declarations of virtual types.

Essential ideas:

- For the realization of a virtual type T, use the type of the primary declaration of T.
- Define a cast method for each primary declaration of a virtual type T. This cast method is overridden in subtypes by overriding declarations.
- Use the cast method for the checking of actual parameters.
- In order to achieve type correctness in the resulting Java programs, introduce further casts.

Example: (Implementation of virtual types)

```
class Vector {
    typedef ElemType as Object;
    void addElement( ElemType e ) { ... }
    ...
}
```

```
class PointVector extends Vector {
    typedef ElemType as Point;
    void addElement( ElemType e ) {
        ...
        e.getX()
        ...
    }
}
```

```
...
PointVector pv;
Point pnt;
...
pv.addElement( pnt );
```

is transformed into

```
class Vector {
    Object cast$ElemType(Object o) {
        return o;
    }
    void check$addElement( Object o ) {
        this.addElement(
            this.cast$ElemType(o) );
    }
    void addElement( Object e ) { ... }
    ...
}

class PointVector extends Vector {
    Object cast$ElemType(Object o) {
        try {
            return (Point)o;
        } catch( ClassCastException c ) {
            throw
                new VirtualTypeCastException(...);
        }
    }
}
```



```
void addElement( Object e$0 ) {  
    Point e = (Point)e$0;  
        ...  
    e.getX()  
    ...  
}  
}
```

```
...  
PointVector pv;  
Point pnt;  
...  
pv.check$addElement (pnt);
```

